Int. J. Heat Mass Transfer. Vol. 15, pp. 2155–2172. Pergamon Press 1972. Printed in Great Britain

A STUDY OF CONVECTIVE HEAT TRANSFER FROM FLAMES

R. CONOLLY and R. M. DAVIES

The Gas Council, Midlands Research Station, Wharf Lane, Solihull, Warwickshire, England

(*Received* 15 *March* 1971 *and in revised form* 10 *January* 1972)

Abstract—A study of the heat transfer from highly dissociated combustion products to a cold surface has been undertaken. Convective heat transfer coefficients have been determined experimentally at the stagnation point of a blunt body immersed in the flames of several common fuel gases burning with pure oxygen. The conditions chosen result in the production of severe temperature gradients which induce correspondingly large changes in species concentrations and thermophysical properties across the boundary layer. Consequently an appreciable proportion of the energy transfer may occur by the diffusion and exothermic recombination of dissociated species.

Theoretical predictions of the heat transfer have been made for the range of conditions studied experimentally. A numerical solution of the appropriate conservation equations has been obtained and a method for the modification of an established low temperature heat transfer relationship has been proposed. Both approaches enable extreme property variations and diffusion–recombination effects to be taken into account when calculating heat flux. Reasonable agreement between experiment and the numerical and semi-empirical predictions has been obtained.

NOMENCLATURE

c,	species concentration;
C_p,	specific heat;
D,	diffusion coefficient;
D_b,	body diameter;
f,	ratio of velocities u/u_e;
G,	mass velocity of approach steam $\rho_e U_\infty$;
Gr,	Grashof number $(\beta g \rho^2 D^3 \Delta T \mu^{-2})$;
g,	ratio of enthalpies I/I_e; acceleration due to gravity in Grashof number;
h,	heat-transfer coefficient $(\dot{q}/\Delta T)$;
H,	enthalpy of fluid excluding kinetic energy term;
H_{chem},	dissociation enthalpy;
I,	enthalpy of fluid including kinetic energy term;
Le,	Lewis number $[D/(\lambda/\rho C_p)]$;
Nu,	Nusselt number (hD_b/λ);
P,	static pressure;
Pr_{eq},	equilibrium Prandtl number $(C_{p,\text{eq}}\,\mu/\lambda_{\text{eq}})$;
Pr_f,	frozen Prandtl number $(C_{p,f}\mu/\lambda_f)$;
$\dot{q}$,	heat flux;
r,	radius;
r_b,	body radius;
Re,	Reynolds number $(D_b G/\mu)$;
St,	Stanton number (h/GC_p);
T,	absolute temperature;
U_∞,	velocity of approach stream;
u,	fluid velocity component parallel to surface;
v,	fluid velocity component perpendicular to surface;
x,	distance from stagnation point measured along the surface;
$\bar{x}$,	transformed x coordinate;
y,	distance measured perpendicularly from surface;
β,	velocity gradient at stagnation point; volumetric coefficient of expansion in Grashof number;
δ	boundary layer thickness;
ρ,	density;
μ,	shear viscosity;
λ,	thermal conductivity;

η, transformed y coordinate.

Superscripts

^{0}, applies to reference state;

$'$, denotes differentiation with respect to η.

Subscripts

e, denotes values at boundary layer edge;

eq, denotes equilibrium values;

f, denotes chemically frozen values;

H, refers to hydrogen atoms;

i, denotes species i;

R, denotes reaction values;

∞, indicates that fluid properties are evaluated for free stream conditions;

w, denotes values at the wall.

Averaged quantities

Average values of thermophysical properties and dimensionless groups are defined in equations 9(a)–(d) and are identified by the bar symbol ($\bar{\ }$)

1. INTRODUCTION

A WIDE range of industrial heating operations now take advantage of the high rates of energy transfer which can be obtained with the use of fuel–oxygen flames. It is evident that the ability to predict accurately the heat fluxes available from these flames would be a valuable asset, particularly in the optimisation of existing processes and in the evaluation of possible new applications.

At the present time it is apparent that the design and operation of gas-fired heat-transfer equipment is based vary largely on accumulated practical experience rather than directly on established scientific principles. This is an inevitable consequence of the dearth of experimental data applicable to high temperature systems, particularly in the fields of natural and forced convective transfer, and of the lack of reliable prediction methods applicable to such environments.

This situation is in striking contrast to that observed under the relatively low temperature conditions which characterise traditional chemical engineering processes. For these, very extensive practical investigations have been carried out and a large number of reliable empirical equations established. These facilitate the confident prediction of heat and mass transfer rates for many common geometrical and fluid combinations and, as such, are of great importance for design purposes. The difficulties which prevent the extension of similar methods to high temperature systems, apart from the shortage of reliable experimental data, stem from the large variations in physical properties which are frequently characteristic of heat transfer from combustion products. These variations hinder the use of empirical equations in the usual forms by introducing difficulties in defining appropriate conditions for the evaluation of the fluid properties, a problem particularly acute where large temperature gradients are present

A further complication in the case of heat transfer from the higher temperature flames resulting from the use of oxygen enrichment is that of dissociation. The presence of dissociated species can result in heat-transfer rates materially higher than predictions from simplified theory. The presence of such species in flames, coupled with the strong dependence of their concentration on temperature, leads to appreciable heat transfer by diffusion–recombination processes and appropriate allowances must be made for these effects when the heat transfer is to be predicted.

The research reported in this paper has been concerned with theoretical and practical studies of forced convective heat transfer from flames at atmospheric pressure. A stagnation point situation has been chosen in order to simplify both experiment and theory. Equilibrium conditions have been assumed throughout and numerical solutions of the appropriate boundary layer conservation equations have been obtained, together with predictions from existing heat transfer relationships modified for application to dissociated combustion products. The theoretical results have then been compared with experimental measurements of the heat transfer between a series of stoichiometric fuel oxygen flames and a heat sink at 400K. Five fuels, namely methane, hydrogen, ethylene, propane and carbon monoxide, have been employed to ensure substantial differences in combustion product composition and transport properties. In addition some measurements have been made using natural gas and air. This work forms part of a systematic study of heat transfer from flames.

2. PREDICTION OF HEAT TRANSFER FROM DISSOCIATED GAS STREAMS

2.1 *Previous work*

Heat transfer from chemically reacting systems has been extensively studied in recent years mostly in relation to atmospheric re-entry problems and the design of rocket thrust chambers. Detailed reviews of this topic have been presented by Lees [1], Chung [2], Bartz [3] and Rosner [4]. Two main lines of approach can be identified. The first of these has been to modify existing low temperature relationships, to take into account both diffusion-recombination phenomena and the property variations within the boundary layer. The second method has been to obtain numerical solutions of the boundary layer conservation equations.

2.1.1 *Modification of low temperature, constant property equations.* In the case of heat transfer to the forward stagnation region of a blunt body of revolution, a solution for laminar, low speed incompressible flow and constant fluid properties has been derived by Sibulkin [5]. This solution takes the form:

$$Nu = 0{\cdot}763\, Pr^{0{\cdot}4}\, D_b(\beta\rho/\mu)^{0{\cdot}5}\,. \tag{1}$$

For a sphere the velocity gradient at the stagnation point β is equal to $3U_\infty/D_b$.

Hence

$$Nu = 1{\cdot}32\, Pr^{0{\cdot}4}\, Re^{0{\cdot}5} \tag{2}$$

or

$$\dot{q} = 1{\cdot}32\, Pr^{0{\cdot}4}\, Re^{0{\cdot}5}\, \lambda\Delta T/D_b \tag{2a}$$

alternatively

$$St = 1{\cdot}32\, Pr^{-0{\cdot}6}\, Re^{-0{\cdot}5} \tag{3}$$

or

$$\dot{q} = 1{\cdot}32\, Pr^{-0{\cdot}6}\, GC_p\Delta T Re^{-0{\cdot}5} \tag{3a}$$

The extension of equations of the above type to chemically reacting systems has been considered in detail by Rosner [4, 6–8]. Using Rosner's approach the stagnation point heat flux to a sphere for a boundary layer in thermochemical equilibrium becomes, by modification of equation (3a).

$$\dot{q} = 1{\cdot}32\, Pr_f^{-0{\cdot}6}\, Re_\infty^{-0{\cdot}5}\, G\Delta H \times [1 + (Le - 1)\Delta H_{\text{chem}}/\Delta H]^{0{\cdot}6}\,. \tag{4}$$

The effects of property variation and energy transfer by diffusion and exothermic recombination have been provided for in two ways in equation (4). Firstly an average Stanton number has been employed such that $\overline{St} = h/G\overline{C}_p$ where $\overline{C}_p = \Delta H/\Delta T$, the use of enthalpy difference reflecting the influence of endothermic dissociation on the effective specific heat of the system. Secondly a correction has been made to the Prandtl number by means of the term in square brackets. The Prandtl number which appears in equation (4) is a "frozen" value, no provision having been made for the influence of endothermicity on specific heat or of diffusion and exothermic recombination on thermal conductivity. The "equilibrium" value of Prandtl number which allows for these effects is in fact equal to the "frozen" value provided that Lewis numbers of diffusing species are unity [9]. Lewis numbers greater than one are to be expected in combustion systems and Rosner has shown that the effective equilibrium Prandtl number in these circumstances is given by

$$[Pr_{eq}]_{av} \doteq [Pr_f]_{av}\,[1 + (Le - 1)\Delta H_{\text{chem}}/\Delta H]^{-1}\,. \tag{5}$$

It has been suggested by Rosner that, where heat transfer from fuel–oxygen flames is concerned, reasonable accuracy is obtained by evaluating the square bracketed correction term using Lewis numbers and dissociation enthalpies based solely on hydrogen atoms. It is also considered sufficient to employ Lewis numbers and frozen Prandtl numbers calculated at wall conditions.

2.1.2 *Numerical solutions.* The pioneering work in the application of numerical methods to heat transfer from reacting systems was carried out by Lees [10]. In solving the appropriate laminar boundary layer conservation equations Lees considered the simplified case where physical property variations through the boundary layer were such that a constant, near unity, Prandtl number could be assumed together with a Lewis number of unity and a constant value of the product of density and viscosity. In addition, implicit within its derivation are the assumptions that chemical equilibrium prevails throughout the boundary layer and that the ratio of free stream to wall enthalpy is large.

A similar approach incorporating certain refinements was made by Fay and Riddell [11] who, by removing the restrictions of unit Lewis numbers and of a constant density–viscosity product, obtained results which were more nearly representative of heat transfer from dissociated air. The equation fitted to their numerical results for equilibrium conditions is

$$\dot{q} = 0{\cdot}763\, Pr_f^{-0{\cdot}6}\, \beta^{0{\cdot}5} (\rho_w \mu_w)^{0{\cdot}1} (\rho_e \mu_e)^{0{\cdot}4} \times \left[1 + (Le^{0{\cdot}52} - 1)\frac{\Delta H_{\text{chem}}}{H_s}\right]\Delta H. \quad (6)$$

Replacing β by $3U_\infty/D_b$ and rearranging gives

$$\dot{q} = 1{\cdot}32\, Pr_f^{-0{\cdot}6} \left(\frac{\rho_w \mu_w}{\rho_e \mu_e}\right)^{0{\cdot}1} Re_\infty^{-0{\cdot}5}\, G\Delta H \times \left[1 + (Le^{0{\cdot}52} - 1)\frac{\Delta H_{\text{chem}}}{\Delta H}\right]. \quad (7)$$

The similarity of equation (7) to equation (4) is readily apparent.

Equations (4) and (7) embody certain simplifying assumptions concerning the transport properties of the fluid which, although reasonable for heat transfer from dissociated air, are invalid for high temperature combustion products. In the Fay and Riddell solution it was assumed that viscosity varies according to the Sutherland formula (i.e. $\mu \alpha T^n$ where $n \sim 0{\cdot}67$). This is certainly a very good approximation in the case of air but is inapplicable where combustion product mixtures are involved. In general a rather stronger dependence on temperature is noted but the variation cannot be accommodated with a constant value of the exponent n. Thus with methane–oxygen combustion products a change in the value of n from 0·94 to 0·80 is required over the range from 600 to 3200 K when a reference temperature of 400 K is selected. Furthermore, in the derivation of both equations (4) and (7) constant values of frozen Prandtl number and Lewis number are assumed. It has been shown by Davies and Toth [12] that for fuel–oxygen combustion products these dimensionless groups vary considerably between adiabatic flame temperature and 400 K, the sink temperature of the present studies. For example, the frozen Prandtl numbers of stoichiometric methane–oxygen products ranged from 0·609 at 3000 K to 0·831 at 400 K, the corresponding figures for H atom Lewis numbers being 3·85 and 7·67 respectively. Finally it must be pointed out that although Rosner does suggest a summation technique for dealing with multicomponent diffusion, the Fay and Riddell equation was derived on the basis of a binary system in which air "atoms" diffused through air "molecules".

In the present investigation both modification of the constant property equations and numerical solution of the boundary layer equations have been undertaken. It is thought that the methods adopted enable a more realistic account to be taken of the thermo-physical properties of fuel–oxygen combustion products than has hitherto been the case.

2.2 *Modified correlation equations for combustion products*

The first modification to be considered is that of an equilibrium Prandtl number incorporating the effect of non-unity Lewis numbers which vary with temperature. An improved method of calculating effective equilibrium Prandtl numbers based solely on H-atom diffusion has been suggested by Davies and Toth [12]. However, a better approach would be the evaluation of Prandtl numbers using equilibrium thermal conductivity values in which more accurate account has been taken of energy transfer by diffusion and recombination. Brokaw [13] has developed a method of calculation whereby the effective total conductivity is determined from the sum of two parts. The "frozen" thermal conductivity arising solely from the molecular collision process, λ_f, is supplemented by an additional quantity called the reacting conductivity, λ_R, which accounts for heat transfer by the diffusion and recombination of all dissociated species.

Thus $$\lambda_{eq} = \lambda_f + \lambda_R. \quad (8)$$

Values of λ_R for the combustion products of methane, hydrogen, ethylene, propane and carbon monoxide burning stoichiometrically with oxygen have been calculated together with the corresponding data for natural gas–air. The other thermodynamic and transport properties* which are necessary for the evaluation of the heat transfer equations have been calculated using the methods and data sources outlined in [12]. The reacting conductivities predicted for the hydrogen–oxygen system have been found to agree closely with the data published by Svehla [14].†

In order to provide for the considerable variations in fluid properties across the boundary layer some form of averaging seems to be essential. Since the variations of thermodynamic and transport properties over the range between flame and sink temperature are non-linear, a weighted average based on the numerical integration of the property with respect to temperature seems reasonable. Thus:

$$\bar{\lambda}_{eq} = (T_e - T_w)^{-1} \int_{T_w}^{T_e} \lambda_{eq}\, \mathrm{d}T \quad \text{and } \overline{Nu} = hD_b/\bar{\lambda}_{eq} \quad (9a)$$

$$\mu = (T_e - T_w)^{-1} \int_{T_w}^{T_e} \mu\, \mathrm{d}T \text{ and } \overline{Re} = GD_b/\bar{\mu} \quad (9b)$$

$$Pr = (T_e - T_w)^{-1} \int_{T_w}^{T_e} \left(\frac{C_{peq}\mu}{\lambda_{eq}}\right) \mathrm{d}T. \quad (9c)$$

When the heat-transfer relationship is in Stanton number form it seems best to average in the same manner as equations (4) and (7).

Thus

$$\overline{St} = \frac{h}{G\overline{C_p}} = \frac{h\Delta T}{G\Delta H}. \quad (9d)$$

Calculated thermophysical properties of stoichiometric combustion products at the flame temperatures measured in the experiments, at the sink temperature of 400 K and average values between flame and sink temperature are given in Table 1.

The various modifications to equations (2) and (3) which have been considered are listed A to F in Table 2. Equations A and B are versions in which all dimensionless groups are averaged. In equations C and D average values of Nusselt, Stanton and Prandtl number are used but the Reynolds number term is evaluated at free stream conditions. Equation D, which is in Stanton number form, is thus closely akin to equations (4) and (7) which have been derived from the work of Rosner and of Fay and Riddell. In equation D the frozen Prandtl number and Lewis number correction terms of equation (4)

---

* Enthalpy, frozen and equilibrium specific heats, density, adiabatic flame temperature, viscosity and frozen thermal conductivity.

† The values of frozen thermal conductivity used by the authors differ from those calculated by Svehla at temperatures below 2000 K. This arises from the use of different input data in the calculation of this property. For the purposes of the present work experimental conductivity data for steam was used up to 1000 K and a curve fitting procedure was used to extrapolate this information to match theoretical predictions at 2000 K and above. The calculations of Svehla were based on the "modified Eucken expression" which, as he himself points out, overestimates experimental data for steam by as much as 16 per cent. The expression used is known to improve in accuracy with increasing temperature and has consequently been retained by the authors for predicting the thermal conductivity of steam at and above 2000 K.

Table 1. Physical properties of combustion products at measured flame temperature, the sink temperature of 400 K and averaged properties between flame and sink temperature

Property		Fuel/Oxidant					
		CH_4/O_2	H_2/O_2	C_2H_4/O_2	C_3H_8/O_2	CO/O_2	NG*/Air
T_e (adiabatic)	K	3052	3073	3216	3097	3004	2224
T_e (measured)	K	3023	3040	3144	3072	2957	2200
$\bar{C}_p$	J/kgK	3646	4765	3750	3595	2370	1435
$\overline{Pr}_{eq}$	—	0·6560	0·7251	0·6373	0·6510	0·6868	0·7337
ρ_e	kg/m^3	$87{\cdot}8 \times 10^{-3}$	$61{\cdot}0 \times 10^{-3}$	$90{\cdot}5 \times 10^{-3}$	$90{\cdot}6 \times 10^{-3}$	0·154	0·152
ρ_w	kg/m^3	0·8126	0·549	0·945	0·899	1·34	0·847
μ_e	Ns/m^2	$87{\cdot}7 \times 10^{-6}$	$87{\cdot}8 \times 10^{-6}$	$89{\cdot}4 \times 10^{-6}$	$88{\cdot}2 \times 10^{-6}$	$83{\cdot}7 \times 10^{-6}$	$67{\cdot}4 \times 10^{-6}$
μ_w	Ns/m^2	$17{\cdot}2 \times 10^{-6}$	$13{\cdot}9 \times 10^{-6}$	$18{\cdot}0 \times 10^{-6}$	$17{\cdot}6 \times 10^{-6}$	$19{\cdot}6 \times 10^{-6}$	$21{\cdot}0 \times 10^{-4}$
$\bar{\mu}$	Ns/m^2	$56{\cdot}5 \times 10^{-6}$	$55{\cdot}6 \times 10^{-6}$	$57{\cdot}9 \times 10^{-6}$	$57{\cdot}4 \times 10^{-6}$	$54{\cdot}4 \times 10^{-6}$	$43{\cdot}8 \times 10^{-6}$
λ_e	W/mK	2·62	3·78	2·72	2·58	0·899	0·233
λ_w	W/mK	$28{\cdot}1 \times 10^{-3}$	$26{\cdot}8 \times 10^{-3}$	$27{\cdot}5 \times 10^{-3}$	$27{\cdot}9 \times 10^{-3}$	$24{\cdot}2 \times 10^{-3}$	$32{\cdot}6 \times 10^{-3}$
$\bar{\lambda}$	W/mK	0·429	0·625	0·520	0·487	0·235	$95{\cdot}0 \times 10^{-3}$

Table 2. Heat transfer coefficients evaluated from modified forms of equations (2) and (3) from equations (4) and (7) and from numerical solutions.

Expressions from which coefficients have been derived	Heat-transfer coefficient (W/m^2K)					
	CH_4/O_2	H_2/O_2	C_2H_4/O_2	C_3H_8/O_2	CO/O_2	NG*/Air
(A) $\overline{Nu} = 1{\cdot}32\overline{Pr}^{0{\cdot}4}\overline{Re}^{0{\cdot}5}$	931	1270	990	885	460	195
(B) $\overline{St} = 1{\cdot}32\overline{Pr}^{-0{\cdot}6}\overline{Re}^{-0{\cdot}5}$	598	740	647	574	366	176
(C) $\overline{Nu} = 1{\cdot}32\overline{Pr}^{0{\cdot}4}Re_\infty^{0{\cdot}5}$	748	1010	797	738	371	157
(D) $\overline{St} = 1{\cdot}32\overline{Pr}^{-0{\cdot}6}Re_\infty^{-0{\cdot}5}$	743	925	805	739	456	219
(E) $\overline{Nu} = 1{\cdot}32\overline{Pr}^{0{\cdot}4}(\bar{\mu}/\mu_e)Re_\infty^{0{\cdot}5}$	482	642	517	480	241	102
(F) $\overline{St} = 1{\cdot}32\overline{Pr}^{-0{\cdot}6}(\bar{\mu}/\mu_e)Re_\infty^{-0{\cdot}5}$	479	588	521	481	296	142
(G) $\overline{St} = 1{\cdot}32\,Pr_{j,e}^{-0{\cdot}6}\left[1+(Le_{f,e,H}^{0{\cdot}5\,2}-1)\dfrac{\Delta H_{\mathrm{chem,H}}}{H_s}\right]\left(\dfrac{\rho_w\mu_w}{\rho_e\mu_e}\right)^{0{\cdot}1} \times Re_\infty^{-0{\cdot}5}$	864	1114	—	862	—	—
(H) $\overline{St} = 1{\cdot}32\,Pr_{f,w}^{-0{\cdot}6}\left[1+(Le_{j,w\,H}^{0{\cdot}5\,2}-1)\dfrac{\Delta H_{\mathrm{chem,H}}}{H_s}\right]\left(\dfrac{\rho_w\mu_w}{\rho_e\mu_e}\right)^{0{\cdot}1} \times Re_\infty^{-0{\cdot}5}$	743	904	—	741	—	—
(I) $\overline{St} = 1{\cdot}32\,\overline{Pr}_{eq}^{-0{\cdot}6}\left(\dfrac{\rho_w\mu_w}{\rho_e\mu_e}\right)^{0{\cdot}1}Re_\infty^{-0{\cdot}5}$	788	950	—	790	—	—
(J) $\overline{St} = 1{\cdot}32\,Pr_{f,w}^{-0{\cdot}6}\left[1+(Le_{w,\mathrm{H}}-1)\dfrac{\Delta H_{\mathrm{chem,H}}}{\Delta H}\right]^{0{\cdot}6}Re_\infty^{-0{\cdot}5}$	794	1087	—	788	—	—
Present numerical solution	544	585	520	493	301	152
Experimental value	521	611	526	465	256	147

$Re_\infty = 300$, $T_w = 400$ K.

*Algerian natural gas, evaporated from liquid storage having a typical composition of 83·9 % CH_4, 11·4 % C_2H_6, 3·4 % C_3H_8, 1·3 % C_4H_{10}.

and (7) have been replaced by an average equilibrium Prandtl number. Equations E and F are modifications of C and D in which an empirical correction factor $\bar{\mu}/\mu_e$ has been added, the significance of which will be explained later.

2.3 *Numerical solution for combustion products*

The important features of the numerical solutions for combustion systems may be seen by reference to the boundary-layer equations for flow over an axially symmetrical blunt nosed body. For thin boundary layers, that is where the boundary-layer thickness is very much smaller than the characteristic dimension of the heated object, these equations are as follows:

global continuity for $\delta \ll r_b$ is defined by

$$\frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial y}(\rho v r) = 0$$

conservation of momentum by

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

and

$$\frac{\partial P}{\partial y} = 0$$

conservation of energy, neglecting thermal diffusion, by

$$\rho u \frac{\partial I}{\partial x} + \rho v \frac{\partial I}{\partial y} = \frac{\partial}{\partial y}\left(\lambda_f \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left\{\frac{\mu}{2}\left(1 - \frac{1}{Pr}\right)\frac{\partial u^2}{\partial y}\right\} + \frac{\partial}{\partial y}\left\{\sum_i \rho D_i (H_i - H_i^0)\frac{\partial c_i}{0y}\right\}$$

where

$$I = H + \tfrac{1}{2}u^2$$

and

$$H = \sum_i c_i (H_i - H_i^0).$$

In the above λ_f is the frozen thermal conductivity of the mixture while the summation term includes the diffusion–recombination effects mentioned previously. In the case of heat transfer from combustion products considered here the energy equation may be modified to avoid the explicit appearance of diffusion terms. This may be accomplished with the use of equilibrium conductivities incorporating the chemical reaction term.

$$\rho u \frac{\partial I}{\partial x} + \rho v \frac{\partial I}{\partial y} = \frac{\partial}{\partial y}\left(\lambda_{\text{eq}} \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left\{\frac{\mu}{2}\left(1 - \frac{1}{Pr}\right)\frac{\partial u^2}{\partial y}\right\}.$$

There is, in the general case, no intrinsic advantage to be obtained from this substitution since λ_{eq} is still dependent on the species diffusivities and the local concentration gradients. In the present instance some saving in computing time could be achieved since values for λ_{eq} had already been calculated.

The boundary-layer equations presented above are a set of non linear partial differential equations. As such they are not easily solved except in restricted circumstances where conditions permit the elimination of certain terms and enable ordinary differential equations to be produced. Such a situation exists close to a stagnation point where, from symmetry considerations, the thermophysical properties must be functions of the displacement from the surface only, and largely independent of x. An approximate solution can then be obtained, provided that a co-ordinate transformation can be found such that the velocity and enthalpy profiles remain similar to themselves over distances corresponding to several boundary-layer thicknesses. This implies that changes occurring in the x direction are small compared with those in the y direction. In such a system it then becomes possible to separate the variables so that ordinary rather than partial differential equations are produced. Thus a transformation from (x, y) to $(\bar{x}, \eta)$ co-ordinates is required such that

$$u(\bar{x}, \eta) = u_e(\bar{x}) f'(\eta)$$

and

$$I = I_e g(\eta).$$

The derivation of the necessary transformations follows the procedure outlined by Fay and Riddell, Lees, Dorrance [15] and others and it can be shown that the application of the transforms

$$\eta = \frac{\rho_e u_e r}{(2\bar{x})^{0\cdot 5}} \int_0^y \frac{\rho}{\rho_e}\,\mathrm{d}y$$

and

$$\bar{x} = \int_0^x \rho_e u_e r^2 u_e\,\mathrm{d}x$$

leads to the equations

$$\left(\frac{C}{Pr} g'\right)' + fg' = \left\{Cf' f''\left(\frac{1}{Pr} - 1\right)\frac{u_e^2}{I_e}\right\}' + \frac{2xf'g}{I_e}\,\frac{\partial I_e}{\partial x}$$

and

$$(Cf'')' + ff'' + \frac{2\bar{x}}{u_e}\,\frac{\partial u_e}{\partial x}\left\{\frac{\rho_e}{\rho} - (f')^2\right\} = 0.$$

Here $C = (\rho\mu/\rho_e\mu_e)$ and $'$ denotes differentiation with respect to η. For the purposes of the present work the value of u_e in the immediately vicinity of the stagnation point was taken from the expression

$$u_e = \tfrac{3}{2}\, U_\infty \sin(x/r_b).$$

The velocity distribution predicted by this equation has been shown to agree with experimental data for flow over spheres for a wide range of Reynolds numbers.

The transformed equations may then be solved by finite difference methods over small increments of η taking various initial guesses for f'' (0) and g' (0) until the conditions at the boundary layer edge are satisfied.

$$f'(\infty) = 1$$

$$g(\infty) = 1$$

$$g'(\infty) = 0.$$

Because simple power law relationships prove to be inadequate for describing the variation of combustion product properties with temperature, this information was fed into the computer program in the form of polynomials in the enthalpy ratio. In order to provide smooth and accurate fitting to the calculated data points, up to 11th order terms were found to be necessary in certain cases. However for the majority of the thermophysical properties a fifth order expression proved to be quite satisfactory.

It is important to note that the derivation of the above set of equations has required that chemical equilibrium prevail throughout the entire system. This should be a reasonable assumption in view of the high temperature levels under consideration and has to some extent been supported by measurements of heat transfer to probes having differing surface characteristics.*

A further point is that the elemental composition is implicitly assumed to remain constant throughout the boundary layer when equilibrium thermal conductivities are specified. This may not always be the case since the diffusion and recombination processes tend to result in slight changes in the proportions of the various elements at different points in the boundary layer.

The solution procedure outlined above permits local enthalpy and velocity profiles to be

---

* It would be expected that the heat transfer from a non-equilibrium system might depend to some extent on the catalytic nature of the surface. This was checked at an early stage in the work by taking measurements of the heat transfer rates to platinum spheres 3 mm dia. both coated with silica and uncoated. After taking into account the slightly increased thermal capacity of the coated sphere in a calibration experiment at 600K, it was found that the sets of data gathered in flames were in reasonable agreement. The coated spheres attracted heat-transfer coefficients some 12 per cent higher than the uncoated versions. This is the reverse of what would be expected from a reduction in catalytic activity and can be attributed to the higher surface temperatures induced by the low thermal conductivity of the silica film. As a result, surface recombination would seem to be absent although this does not, on its own, justify the unequivocal acceptance of the assumption that equilibrium prevails throughout the system.

evaluated in the vicinity of the stagnation point of an axisymmetric body exposed to a particular gas stream. These profiles are initially determined for values of η across the boundary layer and because "similiarity" is assumed in this region they will be virtually independent of x. For the data obtained to have a more practical significance it becomes necessary to introduce the inverse transformation

$$y = \frac{(2\bar{x})^{\frac{1}{2}}}{\rho_e r u_e} \int_0^{\eta} \frac{\rho_e}{\rho} \, \mathrm{d}\eta .$$

Using this relationship and the values of enthalpy and velocity applicable to the boundary layer edge it is then possible to obtain the variation of these quantities with position. Provided the assumption that chemical equilibrium prevails is valid, the thermodynamic state of the boundary layer is then defined and with it the variation of temperature and of all other thermophysical properties.

The heat flux received by the surface may be determined from the following expression

$$q = \lambda_w \left(\frac{\partial T}{\partial y}\right)_w = \frac{\lambda_w}{Cp_w} I_e g'(0) \left(\frac{\partial \eta}{\partial y}\right)_w$$

and the related heat-transfer coefficients deduced from a knowledge of flame and wall temperatures.

3. EXPERIMENTAL TECHNIQUE

The calorimeter used for the measurement of heat flux is illustrated in Fig. 1. A small plug of copper/1 % beryllium alloy was mounted in the nose of a hemispherically ended cylindrical probe, 12·7 mm dia., made from stainless steel. Three sharpened stainless steel pins were used to suspend the plug centrally in a narrow air gap with a minimum of thermal contact area. A thermocouple mounted on the rear surface of the copper element was used to monitor the temperature change. The output from the thermocouple was fed via a suitable resistance network to a UV recorder which had previously been calibrated to enable the temperature history to be obtained from the trace. The freezing points of pure lead and pure tin together with the boiling points of water and of sulphur were used to cover the operating range of the calorimeter. The centre section of the probe was filled with an epoxy resin to prevent circulation of flame gases past the sides of the plug.

The method of operation consisted of briefly

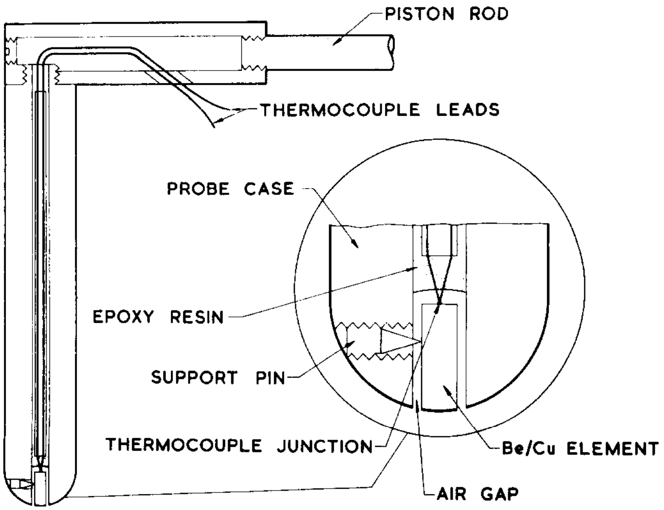

FIG. 1. Sectional diagram showing the construction of the stagnation point heat transfer probe.

inserting the probe into the flame using a pneumatic actuator in conjunction with suitable timing circuits. Measurements were made with the probe axis collinear with that of the flame. The time for a set temperature rise at the rear surface of the plug was recorded. From a knowledge of dimensions and thermal properties it was possible to calculate the heat flux to the front face of the copper plug by solving the one-dimensional transient heat conduction equation.

For the most part experimental measurements involved stoichiometric mixtures with pure oxygen of hydrogen, methane, propane, ethylene and carbon monoxide, which were burnt on a nozzle-mixing glass-working torch. The burner (supplied by the Carlisle Burner Corporation), consisted of a tightly packed bundle of steel capillary tubes through which oxygen was supplied, surrounded by a close-fitting jacket for the fuel gas. There was also a provision for a small pre-mixed flame at the centre of the burner. In operation the burner provided a visible flame of laminar appearance approximately 300 mm in height and 30 mm maximum width. In addition some experiments were also undertaken using premixed stoichiometric natural gas–air and a simple design of burner where the flame was stabilised on a steel plate, 40 mm dia. in which a large number of closely spaced 1·6 mm dia. holes had been drilled. All heat flux measurements were made on the axes of the flames at a height of 12·7 mm above the burner face.

Measurements of gas temperatures were also made at the same height using conventional line reversal equipment. Initially lithium was introduced to produce the spectral line for reversal using a carbon arc background source. Later measurements were made using a sodium salt and a tungsten lamp with a tubular filament which closely approaches a black body source. The seed material was added axially through the small premixed flame in the centre of the burner. A small electromagnet was used to vibrate the steel supply pipe containing a small quantity of the alkali metal chloride. This resulted in sufficient material being carried forward in the gas stream to colour the centre zone of the flame. The addition of seed material as a solid was considered preferable to the atomization of a solution, the latter method giving rise to appreciably lower flame temperatures because of the added moisture. Approach velocities were measured on the flame axis using a water-cooled total head probe. This was designed to have the same dimensions as those of the heat transfer probe, in order to preserve the same flow pattern in each case. The velocities recorded were in agreement with supplementary measurements on the premix burner using a particle track technique.

Significant variations in throughput, giving rise to Reynolds numbers of from 50 to 500, were possible for all fuels. In the case of carbon monoxide, however, a marked tendency for the flame to lift from the burner face had to be countered by the provision of a small annular diffusion flame at the outer edge of the main flame.

A series of heat-transfer measurements was also undertaken in which the probe was cooled from 450K in a uniform laminar air stream at 288K. The object of these latter investigations was to assess the applicability to a hemisphere-cylinder geometry of the expression for velocity gradient ($\beta = 3U_\infty/D_b$) derived for the stagnation point of a sphere. Property variations across the boundary layer were trivial on the chosen experimental conditions and predictions were therefore based on mean film values.

In the range of Reynolds numbers covered (350–4000) it was found that measured coefficients were in close agreement with values calculated from equation (3). The maximum discrepancy amounted to 4 per cent with the theoretical expression generally resulting in a slight underestimation of heat flux. Consequently, in the prediction of stagnation point heat-transfer coefficients to the experimental probe, the proportionality constant in the correlation equation has been taken as 1·32, the value corresponding to a truly spherical geometry.

4. RESULTS

4.1 *Flame temperature measurements*

It was found that the temperature at a point 12·7 mm above the burner on the axis of the flame was constant for a given fuel over the major part of the range of flow rates. Below about one third of the maximum flow a progressive decline in temperature was noted with the lowest value being some 100–200 K below the maximum. Over the normal working range of the burner, which includes most of the experimental data, little dependence on thermal throughput was apparent.

The highest temperatures recorded for each fuel–oxygen combination are listed in Table 1, together with the corresponding calculated adiabatic flame temperatures and the calculated thermophysical properties of the combustion products referred to previously.

4.2 *Heat-transfer coefficients*

Experimental heat-transfer coefficients have been derived from the measured heat fluxes and temperatures. The values corresponding to a free stream Reynolds number of 300 and a probe surface temperature of 400 K are listed in Table 2 for all the five fuel oxygen flames and for the natural gas–air flame. The tabulated coefficients have been estimated from plots of the experimental data. At the chosen Reynolds number, flame temperatures had reached a steady value for all the systems studied.

The corresponding predictions from the various modified forms of equations (2) and (3) and from the numerical solution are also given in Table 2. It is clear that the most generally satisfactory agreement with equipment is provided by equation F and by the numerical method. The agreement with the experimental measurement extends over the whole range of experimental flow rates.

The numerical solution and measured coefficients for the hydrogen–oxygen and methane–oxygen flames are shown in Figs. 2 and 3, the predictions of equation F are not shown on these graphs since they are virtually coincident with the numerical solution. A generalized plot relating the predictions of equation F to measured values is shown in Fig. 4.

5. DISCUSSION

5.1 *Experimental measurements*

The experimental heat-transfer data, having been obtained from the analysis of transient heat flows within the beryllium copper element, are particularly susceptible to uncertainties in the metal properties. Enquiries of the manufacturers revealed that the accuracy of their published data should ensure measured heat fluxes within ± 5 per cent of the true values. In addition it was clear that probable variations between batches were likely to introduce only marginal errors in the experimental results. Heat transfer through the pins supporting the beryllium copper plug and across the annular gap between the plug and the stainless steel case have been estimated to be negligible. It was deemed necessary, however, to check whether or not metallurgical changes could have resulted from the many hundreds of heating cycles involved in the experimental programme. A metallographic examination of the material showed that no alteration of crystalline structure had occurred. In addition, the repetition of some of the runs carried out early in the work indicated that no change in response had been produced.

Possible interactions between free and forced convection mechanisms have been considered. It is evident that the denser boundary layer associated with the cooler surface has an inherent tendency to fall in the hotter gases. The effect is to oppose the flow of the approaching fluid thereby reducing the forced convection component of heat transfer and is, therefore greatest at low gas velocities. Pei [16] has indicated that the effects of natural convection may be completely ignored for $GrRe^{-2} < 0{\cdot}05$ and are less than 10 per cent of the totals for $GrRe^{-2} < 0{\cdot}3$. Calculation shows that the interaction between the two mechanisms can be entirely discounted since the first criterion of

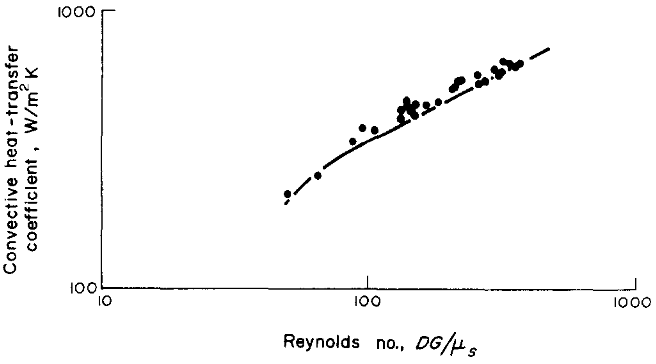

FIG. 2. Convective heat transfer to the stagnation point probe from stoichiometric hydrogen–oxygen combustion products. The continuous line represents the numerical solution of the conservation equations.

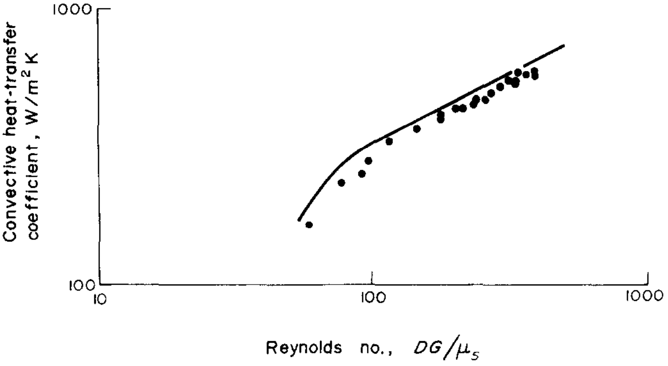

FIG. 3. Convective heat transfer to the stagnation point probe from stoichiometric methane–oxygen combustion products. The continuous line represents the numerical solution of the conservation equations.

$GrRe^{-2} < 0{\cdot}05$ was complied within all but two of the experimental runs. A single data point in each of the series of measurements with hydrogen and ethylene exceeded this limit the respective values being 0·053 and 0·059.

The flame temperatures measured at the set position above the burner generally fall below the calculated adiabatic flame temperatures. This is to be expected since several mechanisms for energy loss are in evidence. Slight losses occur by radiation from the flame gases to the surrounding medium. Snelleman [17] has estimated losses from this effect to be of the order of 20 K in a slightly rich C_2H_2/O_2 flame at the position of maximum temperature. Other minor losses include the effects of incomplete reaction, the temperature of the fuel gases supplied and the difference between the flame and the measured excitation temperatures. The presence of water vapour in the air supply also has an influence on the temperature of the premixed natural gas–air flame, which could amount to a reduction of 15 K compared with the dry air value. The decline in the measured fuel–oxygen flame tem-

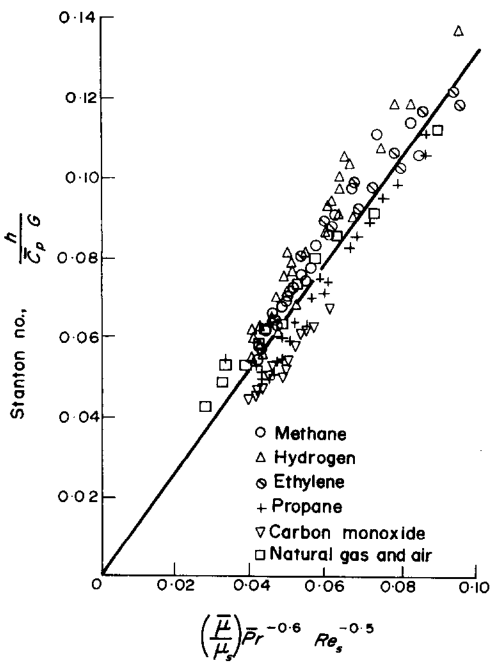

FIG. 4. A generalized plot of the heat transfer data obtained for the combustion products of all five fuel oxygen flames plus the values for the natural gas–air system. The continuous line represents the proposed empirical prediction method.

peratures as the supplies of fuel and oxygen are reduced appears to result from conductive heat losses through the burner, only partially regained by the incoming supplies. It is significant that such effects were absent with the premixed burner which, being of considerably lighter construction, presented less opportunity for irrecoverable heat losses. The effect of these heat losses can be seen quite clearly in Figs. 2 and 3 where both the experimental data points and the results of the numerical integrations exhibit a steady decline as the Reynolds number is reduced. This arises from the changes in free stream conditions and does not reflect any departure from one half power dependence of the heat transfer rate on flow velocity.

It is important to point out that the accuracy of the temperature measurements, (estimated at ±50 K) has a greater influence on the predicted values of heat transfer coefficient than on those derived from experimental observations. An overestimation of flame temperature by this amount leads to an increase in the former of some 7·5 per cent while reducing the latter by approximately 2·5 per cent.

5.2 *Comparison with numerical solutions*

It is evident that the numerical solution procedure used in this work results in generally satisfactory agreement with the experimental data. It was found that the closest agreement was obtained with the methane oxygen and natural gas–air systems and particularly with the latter. This is not unexpected since the smallest property variations occur with this combination and as a result of the lower flame temperature diffusion recombination effects will be slight.

The numerical solution overestimates experimental coefficients for the methane, propane and carbon monoxide flames whereas it underpredicts with ethylene and hydrogen. It seems probable the discrepancy with ethylene is caused by the use of too low a value of flame temperature in deriving the coefficients. Scrutiny of Table 1 shows that the measured flame temperature with this fuel is 72 K less than the theoretical value compared with 20 K to 35 K for the other fuels. This casts some doubt on the experimental value which has been used to calculate the coefficients. If the true temperature is greater than that measured then the coefficients derived from the experiment will be too high and wholly predicted values too low. The underestimation of coefficients for the hydrogen–oxygen flame cannot be attributed to this cause since measured temperatures in this instance are low by the same amount as that encountered in the methane and propane flames.

A possible cause of the "low" prediction for the H_2–O_2 flame is diffusional separation of combustion products in the boundary layer. The use of reacting thermal conductivity data in the solution of the boundary layer equations introduces the assumption that a constant elemental composition is preserved throughout. Nachtsheim [18] has recently shown that this may not be the case. He has pointed out that, as a result of the large differences in the molecular weights and hence in the diffusion coefficents of the species present in hydrogen–oxygen combustion products, some diffusional separation may be expected in systems having severe temperature gradients. Nachtsheim has shown this separation to be significant in the nominally stoichiometric hydrogen–oxygen system at wall and free stream temperatures at 2400 K and 4000 K respectively. Under these conditions a 30 per cent deviation exists in the elemental mass fraction of hydrogen at the wall compared with that at the outer edge of the boundary layer. The effect is strongly temperature dependent and at more modest temperature levels rapidly diminishes. With the same ratio of wall to free stream temperatures as before, but with values of 1824 K and 3040 K, the deviation in elemental composition across the boundary layer results in an increase in the elemental mass fraction of hydrogen near the wall by about 8 per cent. This corresponds to a mixture containing 12·04 per cent of hydrogen by weight instead of the 11·19 per cent which constitutes a stoichiometric mixture with oxygen.

Should this effect be real then the increased hydrogen concentration near to the wall could have an important influence on heat transfer from dissociated systems. It has been shown that, using the numerical prediction procedure, appreciably higher heat transfer rates result when using fuel rich hydrogen–oxygen mixtures at corresponding values of the Reynolds number. It follows that compositional changes resulting from diffusional separation may well be the cause of the measured heat fluxes for the hydrogen–oxygen system being higher than those predicted, although extensive investigations would be necessary before this could be either confirmed or denied.

Although the Fay and Riddell numerical solution was developed for dissociated air the use of the correlation equation has been advocated for combustion products. In applying this equation to combustion systems, uncertainties arise as to how the Prandtl number and Lewis number correction terms should best be evaluated. Two alternative substitutions have been considered in order to compare the predictions of equation (7) with the present numerical solution. These are represented by the expressions G and H in Table 2 which involve the substitution of frozen Prandtl and H-atoms Lewis number evaluated at either free stream or wall conditions. In both cases the term ΔH_{chem} is then restricted to the recombination enthalpy of the hydrogen atoms alone.

Heat transfer coefficients derived from equation (7) according to the schemes described above are compared with corresponding numerical and experimental values for methane, hydrogen and propane flames in Table 2. It is apparent that both forms of substitution result in significantly higher coefficients, the agreement being worse with the version based on free stream values. The discrepancies are such that they cannot be attributed solely to the difference in the allowance for Lewis and Prandtl numbers. This is demonstrated by replacing $Pr_f^{-0\cdot6}$ $[1 + (Le_{\text{H}} - 1)\,\Delta H_{\text{chem, H}}/\Delta H]^{-0\cdot6}$ in equation (7) by $\overline{Pr}_{\text{eq}}^{-0\cdot6}$, which may be considered the better choice where combustion products are involved. The result of this substitution is shown as expression I in the Table and it can be seen that the predictions lie intermediate to those employing the two approaches outlined above.

No definite reason for the disagreement between the measured data for combustion product mixtures and the Fay and Riddell solution can be given at the present time. A positive explanation will require a detailed study of the numerical methods but it is quite possible that the differences may be connected with the dependence of viscosity on temperature in the two systems of dissociated air and combustion products. It will be recalled that the Sutherland viscosity law generally employed for work on air is invalid with the latter. This contention finds some support when a direct comparison of the Fay and Riddell equation and the present numerical method is made for stagnation point heat transfer from air at 3000K. Predictions have been obtained using both methods for surface temperatures in the range 400–1200K. It has been found that while the present method predicts heat fluxes to be slightly lower than those estimated by the more established method the discrepancy amounts to rather less than 12 per cent. This figure is very much less than the margin by which the Fay and Riddell correlation overpredicts where combustion product mixtures are involved.

It seems unlikely that the agreement between experiment and the present numerical solution is fortuitous or, in view of the low temperature experiments, that the velocity gradient could be significantly less than $3U_\infty/D_b$. It is therefore tentatively concluded that the Fay and Riddell correlation cannot be applied to heat transfer from combustion products without appreciable overestimation of coefficients and heat flux.

5.3 *Comparison with modified correlation equations*

Examination of Table 2 shows that modification of Sibulkin's original constant property expression in the manner of equations A–D results in a considerable overestimation of the experimental and numerical coefficients. Of these four equations, A, incorporating averaged Nusselt and Reynolds numbers results in the worst agreement whereas the average Stanton–Reynolds number equation B gives the best fit. Equations C and D involving 'free stream' Reynolds numbers predict coefficients for fuel–oxygen flames intermediate between those of A and B. The values of thermophysical properties given in Table 1 indicate that the use of wholly free stream properties in the correlation equations would result coefficients even higher than those resulting from equation A. By contrast the substitutions based entirely on wall values could lead to coefficients much lower than the experimental and numerical figures.

As mentioned previously equation D is closely akin to the type of relationship proposed by Rosner equation (4) and to the Fay and Riddell correlation. A prediction of coefficients by Rosner's method incorporating a weighted average frozen Prandtl number, H-atom Lewis number evaluated at wall conditions, and ΔH_{chem} corresponding to H atom recombination enthalpy, is also shown in Table 2—equation J. It is clear that both Rosner's expression and form H of the Fay and Riddell equation give results which are of the same order as equation D although all three overestimate experimental data by between 40 and 70 per cent.

A consequence of the disparities which occur using both the immediately obvious modifications of the constant property equations and the other available relationships is that an additional general correction term has been sought in order to obtain a good fit to the experimental data. A parallel, and more scientifically respectable concept, is that an equation fitting the numerical solution should be derived, the form of which is suggested by the original Sibulkin equation and the Fay and Riddell relationship. The most effective means of achieving this state of affairs has been to include the term $(\bar{\mu}/\mu_e)$ in equations C and D. The predictions of the resultant equations (E and F) are shown in Table 2, from which it can be seen that reasonably good agreement is obtained for all fuel–oxygen flames. For the natural gas–air flame the modified Stanton number form, equation F, gives an acceptably good fit to the experimental data whereas the Nusselt form (E) significantly underestimates the coefficients. This effect is puzzling since it might be supposed that the coefficients obtained in the lower temperature flame should be more readily predictable because of the smaller variations in physical properties and a lower degree of dissociation.

In this context it is appropriate to mention the work of Kilham and Cookson [19] on energy transfer from premixed hydrogen air flames and Kilham and Dunham [20] on carbon monoxide flames. These authors employed a similar design of heat flux probe to that used in the present investigations except that it was operated under steady state rather than transient conditions. Kilham and coworkers measured stagnation point heat fluxes in the vicinity of the reaction zone. Particular reference was made to the effects of the condition of the probe surface and the presence or otherwise of atomic and free radical species. In the study of carbon monoxide flames the effects of water and bromine vapour addition were investigated. It was found that heat transfer rates were sensitive to surface condition and the two additives only near to the flame front where non-equilibrium conditions were thought to prevail. At heights greater than 10 mm above the flamefront for the hydrogen flames and 19 mm for the carbon monoxide flames no differences in flux could be detected for a given stoichiometry and fuel input. For the carbon monoxide flames the flux measurements in this downstream region were found to agree within 2 per cent with predictions derived from an expression equivalent, in the notation of the present paper, to:

$$\overline{St} = 1{\cdot}32\, Pr^{-0{\cdot}6}\, Re_{\infty}{}^{-0{\cdot}5}.$$

In the case of the hydrogen–air flames similarly close agreement was also obtained. The flame temperatures involved were 1650–1710K for the CO flame and 1370–1950K for the H_2 flame.

At the distances referred to it may be assumed that the combustion product mixture is virtually in chemical equilibrium and consequently a comparison with the present data for methane–air seems to be reasonable.

The expression quoted above closely resembles equation (D) of Table 2 the only difference being that a Prandtl number based on mean film conditions is preferred to a weighted average value. However, at the temperature levels in question little difference exists between predictions derived from the two equations. It is therefore of interest to note that equation (D) is rather unsuccessful in predicting the heat flux intensities obtained with methane–air flames, resulting in an overprediction of some 50 per cent. No reason for this anomaly can be put forward but it is apparent that equation (D) continues to overpredict where fuel oxygen flames are involved.

The present comparison of empirical prediction and experiment apparently reinforces the proposition that heat flux from high temperature gas streams is best correlated on a Stanton number basis in which enthalpy difference rather than temperature difference may be regarded as the driving force for heat transfer [21]. Such a procedure is likely to reduce the inaccuracies resulting from the employment of theoretical values for high temperature transport properties, the most uncertain of which is thermal conductivity.

The ability of equation F to correlate the experimental results is gratifying but it must be emphasised that the range of experimental conditions over which it has been tested to date is limited. In this respect scrutiny of the thermophysical property data of Table 1 is instructive. From this table it can be seen that the range of Prandtl numbers, viscosity ratios (μ/μ_e) and temperature differences is comparatively small. Significant variations in heat transfer are thus attributed to differences in either mean equilibrium specific heat or mean equilibrium thermal conductivity and the comparisons of Table 2 support this contention. The agreement between predicted and experimental coeffients is strongly dependent on the term ($\bar{\mu}/\mu_e$) by which the Reynolds number is modified. This form of weighting has been tested only over the two temperature ranges 400–2200K and 400–3000K. It therefore seems essential to carry out further investigations over other temperature ranges to test the wider applicability of the correction term used with the present data. Equally it appears necessary to study heat transfer from flames using other geometries for which established low temperature relationships are available.

It should be emphasised that the proposed empirical correlation has little theoretical backing and quite probably an improved version could be obtained with the use of other forms of property averaging. However, it is clear that the present methods provide the basis for making relatively simple estimates of heat transfer from flames under stagnation point conditions. At present the numerical prediction of boundary layer phenomena is of rather limited application, and while flow patterns and heat fluxes can be calculated for simple shapes the method is not available for aerodynamically complex situations. Consequently, it is to be hoped that a similar empirical approach will prove successful in such cases. This possibility is at present being explored.

CONCLUSIONS

A limited measure of success has been achieved in relating the forced convective heat transfer from high temperature flames to the thermophysical properties of their combustion products. It has been shown that numerical solutions of the boundary layer conservation equations can provide predictions of stagnation point heat fluxes which are sufficiently accurate for practical purposes. However, in view of the expensive and time consuming nature of such methods, an empirical modification to a corresponding low temperature relationship has also been obtained to permit more rapid estimates of heat transfer rates to be made.

ACKNOWLEDGEMENTS

This paper is published with the permission of the Gas Council and records part of the work submitted for the degree of Ph.D at the University of Aston in Birmingham. The authors would like to express their thanks to Professor F. M. Page and Dr. R. G. Temple of that University for their numerous helpful discussions and to Mrs. H. E. Toth for carrying out many of the calculations.

REFERENCES

1. L. Lees, *Recent Advances in Heat and Mass Transfer*, p. 161. McGraw-Hill (1961).
2. P. M. Chung, *Advances in Heat Transfer*, Vol. 2, p. 109. Academic Press, New York (1965).
3. D. Bartz, *Advances in Heat Transfer*, Vol. 2, p. 1. Academic Press, New York (1965).
4. D. E. Rosner, Convective heat transfer with chemical reaction, Aeronautical Research Laboratory Report ARL 99 Part I, (1961).
5. M. Sibulkin, Heat transfer near the forward stagnation point of a body of revolution, *J. Aeronaut. Sci.* **19**, 570 (1952).
6. D. E. Rosner, Recent advances in convective heat transfer with dissociative and atom recombination, *Jet Propulsion* **28**, 445, (1958).
7. D. E. Rosner, Enhanced chemical energy transport effects on convective heat fluxes in the hydrogen-oxygen system, *Combustion and Flame* **9**(2), 199 (1965).
8. D. E. Rosner, Effects of diffusion and recombination on convective heat transfer *A.R.S. Jl* **30**, 114 (1960).
9. R. S. Brokaw, The Lewis number. *Progress in International Research on Thermodynamic and Transport Properties*. Academic Press (1962).
10. L. Lees, Laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds, *Jet Propulsion* **26**(4), 259 (1956).
11. J. A. Fay and R. F. Riddell, Theory of stagnation point heat transfer in dissociated air, *J. Aeronaut. Sci.* **25**, 73 (1958).
12. R. M. Davies and H. E. Toth, Thermodynamic and transport properties for fuel oxygen combustion systems. Fourth Symposium on Thermophysical Properties, ASME, p. 350 (1968).
13. R. S. Brokaw, Thermal conductivity of gas mixtures in chemical equilibrium *J. Chem. Phys.* **32**, 1005 (1960).
14. R. A. Svehla, Thermodynamic and transport properties for the hydrogen oxygen system, NASA SP-3011 (1964).
15. W. H. Dorrance, *Viscous Hypersonic Flow*, McGraw-Hill (1962).
16. D. C. T. Pei, Heat transfer from spheres under combined forced and natural convection, *Chem. Engng. Symp.* **61**(59), 57 (1965).
17. W. Snelleman, A flame as a standard of temperature, Ph.D.Thesis, University of Utrecht (1965).
18. P. R. Nachtsheim, Proceedings of the Heat Transfer and Fluid Mechanics Institute, University of California (June 1967).
19. J. K. Kilham and P. G. Dunham, Energy transfer from carbon monoxide flames. *11th Symposium (International) on Combustion*, p. 899. The Combustion Institute, Pittsburgh (1967).
20. J. K. Kilham and R. A. Cookson, *9th Symposium (International) on Combustion*, p. 257. Academic Press (1963).
21. D. E. Rosner, Discussion of a paper by R. N. Meroney and W. H. Geidt, *J. Heat Transfer* **89**, 205 (1967).

UNE ETUDE DE TRANSFERT THERMIQUE PAR CONVECTION DANS LES FLAMMES

Résumé—On entreprend une étude du transfert thermique entre des produits de combustion fortement dissociés et une surface froide. Les coefficients de transfert thermique par convection ont été déterminés expérimentalement au point d'arrêt d'un corps émoussé, immergé dans des flammes de plusieurs gaz combustibles usuels brûlant dans de l'oxygène pur. Les conditions choisies résultent de la production de forts gradients de température qui induisent de grands changements dans les concentrations en espèces et dans les propriétés thermophysiques à travers la couche limite. En conséquence une part appréciable du transfert d'énergie peut être due à la diffusion et à la recombinaison exothermique d'espèces dissociées.

Des estimations théoriques du transfert thermique ont été faites pour le domaine de conditions étudiées expérimentalement. Une solution numérique des équations de conservation appropriées a été obtenue et une méthode pour la modification d'une loi sur le transfert thermique à basse température a été proposée. Les approches permettent de tenir compte des variations de propriété extrêmes et des effets de la diffusion-recombinaison dans le calcul du flux thermique. Il existe un accord raisonnable entre l'expérience et les estimations numériques semi-empiriques.

UNTERSUCHUNG DES KONVEKTIVEN WÄRMEÜBERGANGS AN FLAMMEN

Zusammenfassung—Eine Untersuchung des Wärmeübergangs von hochdissozierten Verbrennungsprodukten an einer kalten Oberfläche wurde vorgenommen. Für den Staupunkt eines stumpfen Körpers, in Flammen verschiedener, herkömmlicher Brenngase, die mit reinem Sauerstoff verbrannt werden, sind die Wärmeübergangskoeffizienten experimentell bestimmt worden. Unter den gewählten Bedingungen ergeben sich grosse Temperaturgradienten, die entsprechend grosse Änderungen in der Konzentration und den thermophysikalischen Eigenschaften quer zur Grenzschicht hervorrufen. Folglich kann eine beträchtliche Menge der Energieübertragung durch Diffusion und exotherme Rekombination der dissoziierten Arten auftreten. Für eine Reihe von Bedingungen, die experimentell untersucht wurden, konnten theoretische Voraussagen der Wärmeübertragung gemacht werden. Aus den entsprechenden Erhaltungsgleichungen wurde eine numerische Lösung erhalten und eine Methode zur Modifikation einer Wärmeübergangsbeziehung bei festgelegter niedriger Temperatur worde vorgeschlagen. Beide Näherungen sind sehr flexibel und berücksichtigen bei der Berechnung des Wärmestromes Diffusions- und Rekombinationseffekte. Zwischen Experiment und numerischer und halbempirischer Lösung wurde eine vernünftige Übereinstimmung erhalten.

ИССЛЕДОВАНИЕ КОНВЕКТИВНОГО ТЕПЛООБМЕНА ПЛАМЕН

Аннотация—Проведено исследование теплоотдачи от сильно диссоциированных продуктов горения к холодной поверхности. Коэффициенты конвективного теплообмена определены экспериментально в критической точке затупленного тела, помещенного в пламя нескольких обычных топливных газов, горящих в чистом кислороде. В рассматриваемых условиях возникают большие градиенты температуры, которые соответственно вызывают значительные изменения концентраций материалов и теплофизических свойств по толщине пограничного слоя. Следовательно, перенос энергии может осуществляться диффузией и при экзотермической рекомбинации диссицированных компонентов.

Проведен теоретический расчет теплообмена в диапазоне условий эксперимента. Получено численное решение соответствующих уравнений сохранения, и предложен метод модификации установленной температурной зависимости теплообмена. Оба метода позволяют учесть влияние теплофизических свойств и диффузионных параметров при расчете теплового потока. Получено удовлетворительное соответствие между экспериментом и численными и полуэмпирическими расчетами.